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TISSERAND ON THE PRESENT CONDITION OF THE THEORY OF THE MOON.*

While preparing his "Traité de Mécanique Céleste" Tisserand has given us many valuable notes on subjects in celestial mechanics. One of the most interesting of these is this "Note sur l'État Actuel de la Théorie de la Lune," in which he has followed pretty closely Professor Newcomb's "Researches on the Motion of the Moon," but has presented, however, some important additions.

Although when Hansen's Tables of the Moon were published it was thought that the complicated problem of the Moon's motion was definitively solved, and although these tables seemed to satisfy very exactly the observations from 1750 to 1850, the period of accurate observations which Hansen had at his disposal to examine, yet soon after 1860 it became evident that the tables were in error, and that the errors were increasing.

The question how Hansen's Tables satisfied observations before 1750 was taken up by Newcomb in his Researches on the Motion of the Moon. All the available material was collected, and carefully and ably discussed by him. He finds from the eclipses of the moon given in the Almagest of Ptolemy the following corrections to the mean longitudes of Hansen's Tables:

Epoch.	*Corr. to Tables.*
— 687	— 11′ ± 4′
— 381	— 27 ± 5
— 189	— 20 ± 3
+ 134	— 16 ± 4

The eclipses observed by the Arabs give

Epoch.	*Corr. to Tables.*
850	— 3′.8 ± 2′.4
927	— 1 .6 ± 1 .7
986	— 4 .5 ± 1 .3

For the epochs after 1620 Newcomb finds the following corrections:

1625	+ 50″ ± 13″	1775	0″ ± 1″
1650	+ 39 ± 5	1800	0 ± 1
1675	+ 32 ± 1	1825	0 ± 1
1700	+ 21 ± 1	1850	0 ± 1
1725	+ 7 ± 1	1875	— 8 ± 1
1750	0 ± 1		

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* Note sur l'État Actuel de la Théorie de la Lune. Par M. F. Tisserand. Bulletin Astronomique, November, 1891.

In endeavoring to account for these quite inadmissible errors Tisserand considers the calculation of the solar perturbations, the calculation of the inequalities of long period, and the numerical determination of the constants. The coefficients of the inequalities arising from the action of the sun were determined by Hansen and Delaunay by quite different methods, and were shown by Newcomb to be in practical agreement.

For the two inequalities of long period arising from the action of Venus the values adopted by Hansen are

$$V_1 = + 15''.34 \sin (- g - 16\, g' + 18\, g'' + 33^\circ\, 36'), \text{ period of 273 years,}$$
$$V_2 = + 21\ .47 \sin (8\, g'' - 13\, g' + 4^\circ\, 44'), \qquad \text{period of 239 years ;}$$

while the values found theoretically by Delaunay are

$$V_1 = + 16''.34 \sin (- g - 16\, g' + 18\, g'' + 35^\circ\, 16'.5),$$
$$V_2 = + \ 0''.27 \sin (8\, g'' - 13\, g' - 41^\circ\, 48'),$$

where g, g', g'' are the mean anomalies of the Moon, the Earth, and Venus respectively.

Hansen's V_1 was found by theory, while V_2 was determined by him empirically so as to satisfy observations from 1750 to 1850. Also, Hansen has adopted 12″.17 for the secular acceleration s, though the theoretical value determined by Adams and Delaunay is 6″.18, the former value being supposed better to represent the ancient eclipses.

When Hansen's V_2 is added to the tabular corrections they assume the following values, failing to satisfy the observations from 1750 to 1850:

1625	+ 33″	1775	+ 21″
1650	+ 18	1800	+ 15
1675	+ 15	1825	+ 2
1700	+ 16	1850	— 11
1725	+ 16	1875	— 28
1750	+ 19		

Since the presence of the term V_2 furnished Hansen with an erroneous value of the mean motion, a correction δn is needed; also, corrections δs and δe should be given to the secular acceleration and the longitude of the epoch. But as an examination of the table just presented shows that there is no system of values of δn, δs, and δe that will make the corrections vanish, we have to assume that the present theory is incapable of representing the observations from 1625 to 1875, and all that can be done is to introduce an empirical term, such as

$$R = A \sin at + B \cos at,$$

and then seek to determine the quantities δn, δs, δe, A, B, and a, so as to satisfy the observations.

Newcomb has used for a a value such that the period of R is 273 years, which is the period of V_1, and the table last given furnishes eleven equations of condition for determining δe, δn, A, and B in terms of δs. Tisserand solves the equations four times, using for the period of R 391, 273, 209, and 170 years; and gives the residuals when δs is left undetermined, and when it is put equal to — 6″.0; that is, when s has its theoretical value. The residuals corresponding to the period 170 years of R are larger than the modern observations would admit. The corrections to Hansen's Tables for 1889.0 are now calculated with the first three values of the period of R, and are found to be

Period of R.	*Corr.*	*Corr.*
391 years	$-20''.1 + 0''.17\ \delta s$	$-21''.1$
273 years	$-15.1 + 0.38$	-17.4
209 years	$-7.8 + 0.69$	-11.9

the values in the second column being those for which $\delta s = -6''.0$. The correction — 17″.4 corresponding to the period 273 years agrees exactly with the observed correction. Then with this period, using the corresponding values of δe, δn, A, and B, are calculated corrections C and C' to Hansen's Tables for the years 1620–1888, both for $s = 12''.17$ and for the theoretical value $s = 6''.18$. Under O are given the corrections to the tables found from observations.

	C	O	C'	$O - C'$
1620	+ 50″	+ 53″	+ 46″	+ 7″
30	+ 48	+ 48	+ 46	+ 2
40	+ 45	+ 43	+ 44	— 1
50	+ 42	+ 39	+ 42	— 3
60	+ 38	+ 36	+ 39	— 3
70	+ 34	+ 33	+ 35	— 2
80	+ 30	+ 30	+ 30	0
1690	+ 24	+ 26	+ 25	+ 1
1700	+ 20	+ 21	+ 20	+ 1
10	+ 15	+ 15	+ 15	0
20	+ 11	+ 9	+ 11	— 2
30	+ 7	+ 5	+ 6	— 1
40	+ 1	+ 2	+ 1	+ 1
50	+ 1	0	0	0
60	0	0	— 1	+ 1
70	— 1	0	— 2	+ 2
80	— 2	0	— 2	+ 2
90	— 1	0	— 1	+ 1

	C	O	C'	$O-C'$
1800	0″	0″	0″	0″
10	0	0	+ 1	− 1
20	+ 1	0	+ 2	− 2
30	+ 1	0	+ 2	− 2
40	+ 1	0	+ 2	− 2
50	0	0	+ 1	− 1
50	− 0.1	+ 0.7	+ 0.7	0.0
52	− 0.5	+ 1.3	+ 0.2	+ 1.1
54	− 1.0	+ 1.4	− 0.3	+ 1.7
56	− 1.4	+ 1.2	− 0.9	+ 2.1
58	− 1.9	+ 1.9	− 1.4	+ 3.3
60	− 2.3	+ 2.3	− 1.9	+ 4.2
62	− 2.9	+ 2.2	− 2.6	+ 4.8
64	− 3.5	+ 0.1	− 3.3	+ 3.4
66	− 4.2	− 2.3	− 4.1	+ 1.8
68	− 4.9	− 4.0	− 5.0	+ 1.0
70	− 5.7	− 5.4	− 5.9	+ 0.5
72	− 6.5	− 7.5	− 6.9	− 0.6
74	− 7.4	− 9.1	− 8.0	− 1.1
76	− 8.3	− 9.6	− 9.1	− 0.5
78	− 9.3	− 9.0	− 10.2	+ 1.2
80	− 10.3	− 10.3	− 11.4	+ 1.1
82	− 11.3	− 12.6	− 12.7	+ 0.1
84	− 12.3	− 14.8	− 14.0	− 0.8
86	− 13.4	− 15.4	− 15.4	0.0
1888	− 14.6	− 16.9	− 16.8	− 0.1

The computed corrections to the tables agree well with the observed, but there would seem to be another inequality having a coefficient of $2''$ or $3''$.

Having recourse to the eclipses of Ptolemy and those of the Arabs for the determination of δs, the most probable value is found to be $-5''.87$, or $s = 6''.3$, very nearly the theoretical value.

The most important addition made in this paper by Tisserand to the theory of the Moon is to point out that with the addition of an empirical term we can represent its motion by the theoretical value of the secular acceleration as well as by any. Thus, we can find as good places by using pure theory as by having regard to the friction of the tides, and supposing a diminution in the velocity of rotation of the Earth and an apparent acceleration of the movement of the Moon. Such a change in the Earth's velocity of rotation would be so small that it could hardly be measured, and though sustained by theory we cannot tell whether the change really takes place or whether it is counteracted in some way that we do not know.

The chronological eclipses, which would require s to have the value $12''$, are rejected by Newcomb and Tisserand, as the accounts of them are vague and untrustworthy.

The inequality of long period,

$$R = A \sin at + B \cos at,$$

which is needed to determine the mean motion with precision, and is introduced as an empirical term, remains to be accounted for theoretically. The combinations that can be made of the mean longitudes of the Moon, the Earth, and a planet, and those of their nodes and perihelions are very numerous, and there may be, besides V_1, and the small inequality arising from the action of Jupiter discovered by Neison, other inequalities having sensible coefficients. Also, the value of V_1, may not be correctly determined. But Tisserand thinks that in the end theory will triumph, and the law of gravitation will completely explain the Moon's motion.

ASAPH HALL, JR.

NOTE.

The principal formula demonstrated in the article "On certain space and surface integrals," pp. 61–63 of this volume, had already been given by J. Somoff. In the more general case of oblique co-ordinates it appears in his memoir "*Moyen d'exprimer directement en coordonnées curvilignes quelconques, orthogonales ou obliques, les paramètres différentiels du premier et du second ordres et la courbure d'une surface*," p. 14, *Memoirs St. Petersburg Academy*, 7 series, Vol. VIII. (1865), and is repeated in his Theoretical Mechanics with applications to several special cases. It is found on p. 13 of the second volume of Professor Ziwet's translation of the work into German, *Theoretische Mechanik*, Leipzig, Teubner, 1879. The information contained in this note was kindly supplied to the author by Professor Ziwet.

THOMAS S. FISKE.

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